VIRILE AGITUR

Knox Grammar School

2016

TRIAL HSC
Examination

Name: ____________________

Teacher: ____________________

Year 12 Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using blue or black pen only
- Board approved calculators may be used
- The official BOSTES reference sheet is provided
- In questions 11 – 16 show relevant mathematical reasoning and/or calculations

Teachers:

Vuletich M.
Mulray I.
Ruff E.
Willcocks A.

Section I ~ Pages 1 – 5

- 10 marks
- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Use the Multiple Choice Answer Sheet

Section II ~ Pages 6 – 14

- 90 marks
- Attempt Questions 11 – 16
- Allow about 2 hours 45 minutes for this section
- Answer each question in a separate writing book

Examiner: Ms Ruff

Write your Name, your Board of Studies Student Number and your Teacher's Name on the front cover of each answer booklet

This paper MUST NOT be removed from the examination room.
Number of Students in Course: 98

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Section I

10 marks
Attempt questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10.

Question1

What is the value of r, correct to 3 significant figures, if $\frac{4}{3}\pi r^3 = 200$?

(A) 1.40

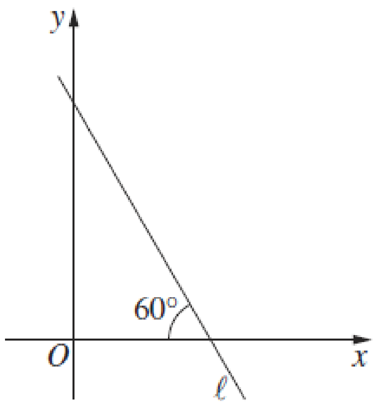
(B) 3.62

(C) 3.628

(D) 3.63

Question 2

The diagram shows the line *l*.



What is the slope of the line *l*?

(A) $-\sqrt{3}$

(B) $-\frac{1}{\sqrt{3}}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\sqrt{3}$

Question 3

What is the value of $\sum_{k=1}^{4}(-1)^k k^2$?

(A) -30

(B) -10

(C) 10

(D) 30

Question 4

Which of the following statements is true for the equation $1-4x-5x^2=0$?

(A) No real roots.

(B) One real root.

(C) Two real and rational roots.

(D) Two real and irrational roots.

Question 5

Evaluate $\lim_{h\to 4}\dfrac{4-h}{16-h^2}$.

(A) 0

(B) $\dfrac{1}{8}$

(C) $\dfrac{1}{4}$

(D) 4

Question 6

In the diagram *AE* is parallel to *BD*, *A*B = *x* cm , *BC* = 3*x* cm and *EC* = 24 cm.

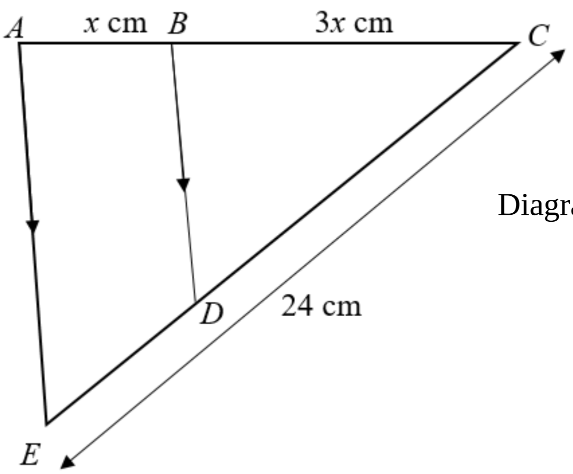


Diagram not to scale

The length of *DC* is:

(A) 6 cm

(B) 8 cm

(C) 12 cm

(D) 18 cm

Question 7

If $y = (x+1)^3(x-3)$ then $\frac{dy}{dx}$ is equal to:

(A) $-2(x+1)^2(x+4)$

(B) $4(x+1)^2(x-2)$

(C) $2(x+1)^2(5-x)$

(D) $2(x+1)^2(x-1)$

Question 8

What is the value of $\int \frac{\sin x}{\cos x} dx$?

(A) $\sec^2 x + C$

(B) $\frac{1}{2}\tan^2 x + C$

(C) $\log_e \cos x + C$

(D) $\log_e \sec x + C$

Question 9

How many solutions of the equation $\cos 2x(\tan x - 1) = 0$ lie in the domain $0 \le x \le \pi$?

(A) 2

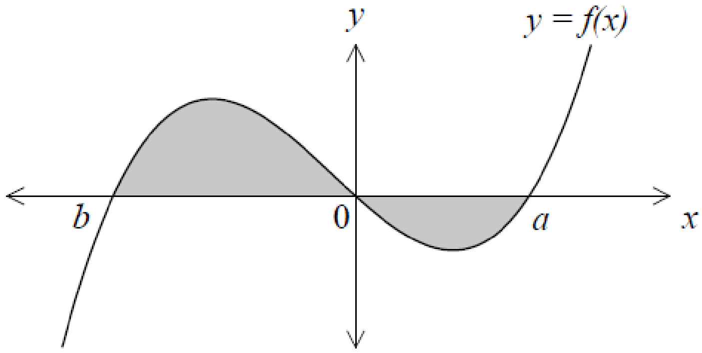
(B) 3

(C) 4

(D) 5

Question 10

Which of the following correctly finds the shaded area in this diagram?



(A) $\int_b^a f(x)dx$

(B) $\int_b^0 f(x)dx + \int_0^a f(x)dx$

(C) $\int_b^0 f(x)dx - \int_0^a f(x)dx$

(D) $\int_0^a f(x)dx - \int_b^0 f(x)dx$

End of Section I

Section II

90 marks
Attempt questions 11 – 16
Allow about 2 hours 45 minutes for this section
Answer each question in a separate writing booklet. Extra writing booklets are available.
All necessary working should be shown in every question.

Question 11 (15 marks) Use a SEPARATE writing booklet. **Marks**

(a) Factorise fully $8x^3 + 27$. **1**

(b) Write $\dfrac{\sqrt{7}}{\sqrt{7}+2}$ with a rational denominator. **2**

(c) Find the coordinates of the focus for the parabola $y^2 = -8x + 24$. **2**

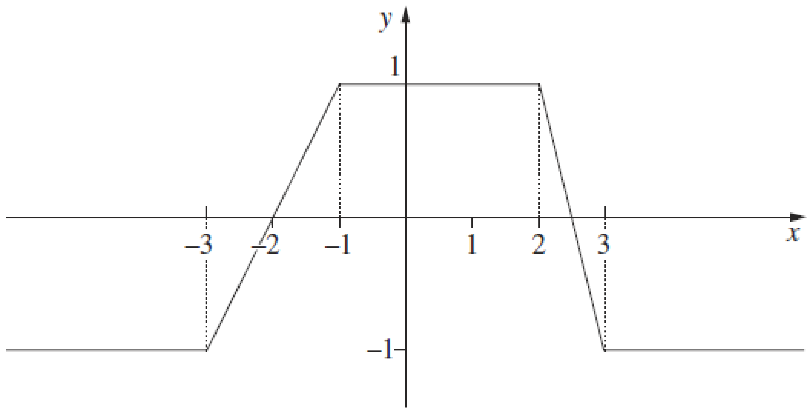
(d) Differentiate $\dfrac{\cos x}{x^2}$ with respect to x. **2**

(e) Differentiate $(e^x + x)^5$. **2**

(f) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sin\frac{x}{2}\,dx$. **3**

(g) Find $\displaystyle\int \frac{1}{x^2}\,dx$. **2**

(h) The diagram shows the graph $y = f(x)$. **1**



What is the value of a, where $a > 0$, so that $\displaystyle\int_{-a}^{a} f(x)dx = 0$?

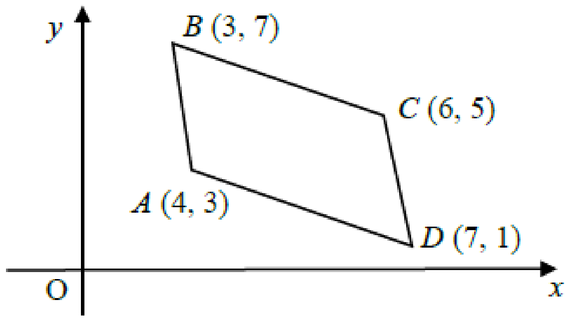
End of Question 11

Question 12 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) Find the equation of the normal to the curve $y = 4\sqrt{x}$ at the point $(9,\ 12)$. **3**

(b) The quadratic equation $5x^2 + 3x - 10 = 0$ has the roots α and β. Find $\alpha^2 + \beta^2$. **2**

(c) In the diagram below the points $A(4,3)$, $B(3,7)$, $C(6,5)$ and $D(7,1)$ form a parallelogram, *ABCD*.



(i) Show that the equation of the line *AD* is $2x + 3y - 17 = 0$. **1**

(ii) Find the exact length of BC. **1**

(iii) Find the perpendicular distance from *C* to the line *AD*. **2**

(iv) Hence, or otherwise, find the area of parallelogram *ABCD*. **1**

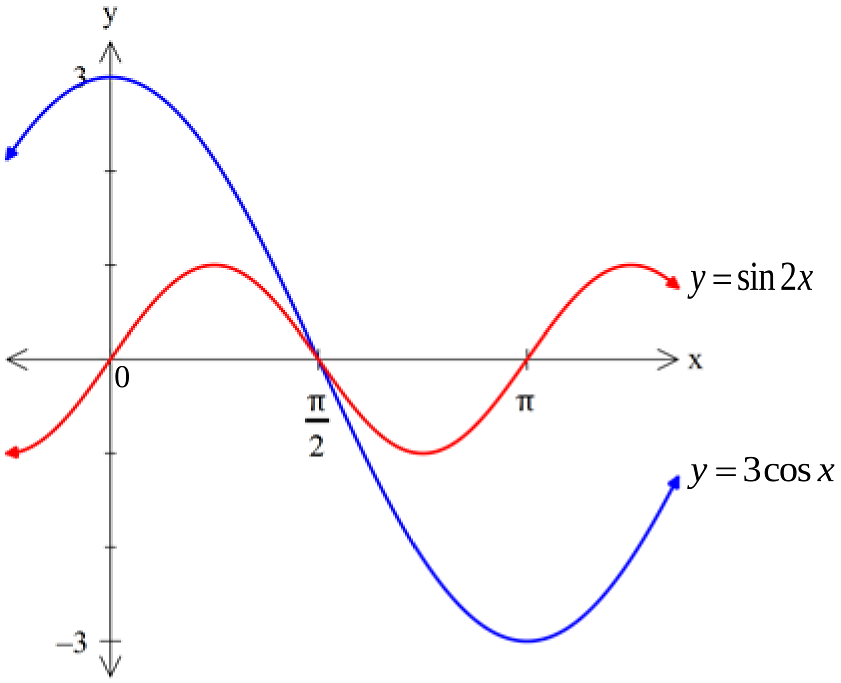
(d) Use Simpson's rule with three function values to find an approximation for $\int_2^6 \frac{x}{\ln x}\,dx$, **2**

giving your answer correct to 1 decimal place.

Question 12 continues on next page

Question 12 (continued)

(e) The diagram below shows the curves $y = 3\cos x$ and $y = \sin 2x$. **3**



Find the area between the curves $y = 3\cos x$ and $y = \sin 2x$ in the domain $0 \leq x \leq \pi$.

End of Question 12

Question 13 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) (i) Find the domain and range of the function $f(x)=\sqrt{9-x^2}$. **2**

(ii) On a number plane, shade the region where the points $(x,\ y)$ satisfy both of the inequalities $y \le \sqrt{9-x^2}$ and $y \ge x$. **2**

(b) Initially there are 1200 individuals in a population. After t years the number of individuals in the population is given by $N(t)=1200e^{kt}$ for some constant $k>0$. After 18 years there are 3000 individuals in the population.

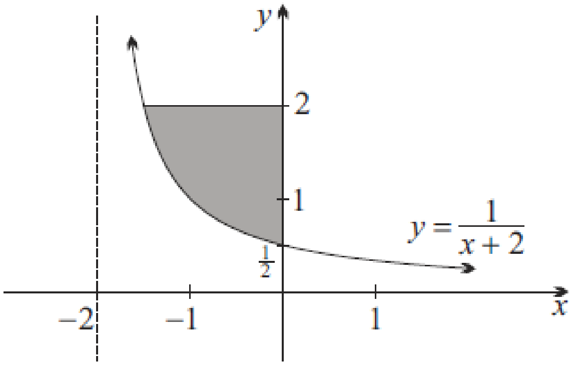
(i) Show that $k=0.0509$ correct to 4 decimal places. **2**

(ii) Find the number of years required for the number of individuals to increase from 1200 to 2400. Give your answer correct to the nearest year. **2**

(c) The derivative of a function $f(x)$ is $f'(x)=8x+3$. The line $y=4-5x$ is a tangent to the graph of $f(x)$. **3**

Find the function $f(x)$.

(d) The diagram below shows the curve $y=\dfrac{1}{x+2}$ for $x>-2$.



(i) Show that $x^2=\dfrac{1}{y^2}-\dfrac{4}{y}+4$. **1**

(ii) Calculate the exact volume of the solid of revolution formed when the shaded region bounded by the y-axis, the line $y=2$ and the curve is rotated about the y-axis. **3**

End of Question 13

Question 14 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) Consider the function $f(x) = x^3 + 6x^2 - 135x$.

(i) Find the coordinates of the stationary points of the curve $y = f(x)$ and determine their nature. **3**

(ii) Show that there is a point of inflexion at $x = -2$. **1**

(iii) Sketch the graph of $y = f(x)$ showing all important features. **2**

(b) The rate (R) at which greenhouse gases are released into the atmosphere from a town in tonnes/hour is given by $R = 20 + \dfrac{100}{(1+2t)^2}$, where t is in hours.

(i) At what rate are the greenhouse gases released initially? **1**

(ii) What is the rate at which greenhouse gases are released as time increases indefinitely? **1**

(iii) Without using calculus, sketch a graph of R as a function of time. **1**

(iv) How much gas was released into the atmosphere in the first 2 hours? **2**

(c) The velocity, $\dot{x}$, in m/s of a particle moving in a straight line is given by:

$$\dot{x} = 3 - \frac{9}{t-2} \text{ for } t > 2,$$

where t is the time in seconds.

(i) In which direction is the particle travelling when $t = 3$? **1**

(ii) Find the time when the particle changes direction. **1**

(iii) Hence, or otherwise, find the distance travelled by the particle between $t = 3$ and $t = 7$. Give your answer correct to 2 decimal places. **2**

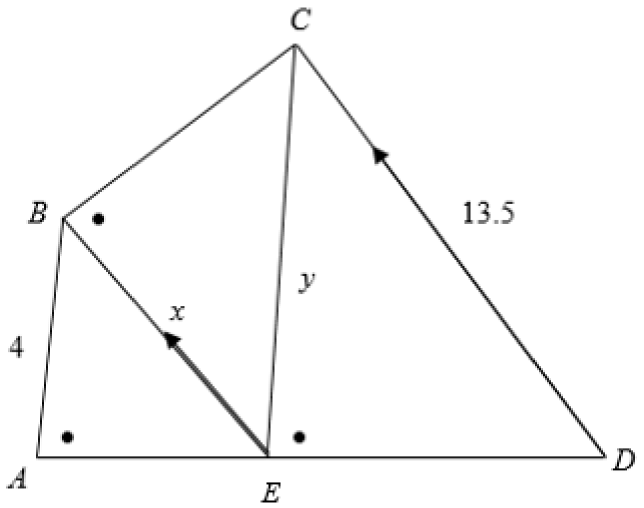
End of Question 14

Question 15 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) (i) Sketch the graph $y = |2x - 3|$. **1**

(ii) Using the graph from part (i) or otherwise, find the values of m for which the equation $|2x-3| = mx + 1$ has exactly 1 solution. **2**

(b) In the diagram BE is parallel to CD, $AB = 4$, $CD = 13.5$, $BE = x$, $CE = y$ and $\angle BAE = \angle EBC = \angle CED$.



Not to scale

(i) Prove that $\triangle CED \,|||\, \triangle BAE$. **2**

(ii) Prove that $\triangle CEB \,|||\, \triangle EBA$. **2**

(iii) Using parts (i) and (ii) show that 4, x, y, 13.5 are the first 4 terms of a geometric series. **1**

(iv) Hence find the values of x and y. **2**

Question 15 continues on next page.

Question 15 (continued)

(c) Wilfred borrows \$500 000 in order to purchase a unit. Reducible interest is charged at 4.8% per annum, calculated monthly. At the end of the first month Wilfred makes a loan repayment of \$*M*. At the end of each subsequent month Wilfred makes a loan repayment that is 1% more than the previous repayment.

(i) Show that Wilfred's balance at the end of the second month is: **1**

$$A_2 = 500000 \times 1.004^2 - M(1.004 + 1.01)$$

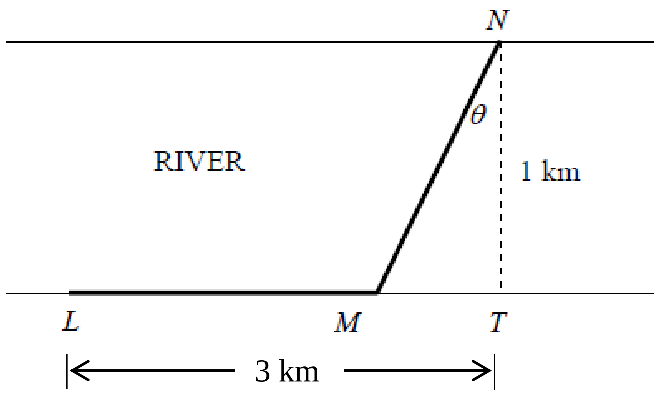
(ii) Given $A_4 = 500000 \times 1.004^4 - M(1.004^3 + 1.004^2(1.01) + 1.004(1.01)^2 + 1.01^3)$, write an expression for the amount owing at the end of 5 years. **1**

(iii) What will Wilfred's initial repayment, \$*M*, need to be in order for the balance of his loan to be \$400 000 at the end of 5 years? **3**

End of Question 15

Question 16 (15 marks) Use a SEPARATE Writing Booklet. **Marks**

(a) A cable link is to be constructed between two points L and N, which are situated on opposite banks of a river of width 1 km. L lies 3 km upstream from N. It costs three times as much to lay a length of cable underwater as it does to lay the same length overland. The following diagram is a sketch of the cables where θ is the angle NM makes with the direct route across the river.



Not to scale

(i) Show $MN = \sec\theta$ and $MT = \tan\theta$. **1**

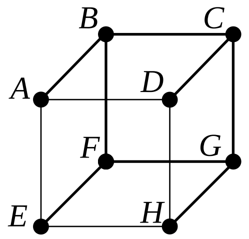
(ii) If segment LM costs c dollars per km, show the total cost (T) of laying the cable from L to M to N is given by: **2**

$$T = 3c - c\tan\theta + 3c\sec\theta.$$

(iii) At what angle, θ, should the cable cross the river in order to minimise the total cost of laying the cable? **3**

Question 16 continues on next page

(b)



In the diagram above, the vertices on the top face of a cube are A, B, C and D and the corresponding vertices on the bottom face of the cube are E, F, G and H.

A robotic device travels along the edges of this cube always starting at A and **never** repeating an edge. This defines a *trail* of edges. For example, *ABFE* and *ABCDAE* are trails, but *ABCB* is not a trail. The number of edges is called its *length*.

At each vertex, the robotic device must proceed along one of the edges that has not yet been traced, if there is one. If there is a choice of untraced edges, the following probabilities for taking each of them apply:

I. If only one edge at a vertex has been traced and that edge is vertical, then the probability of the robot taking each horizontal edge is $\frac{1}{2}$.

II. If only one edge at a vertex has been traced and that edge is horizontal, then the probability of the robot taking the vertical edge is $\frac{2}{3}$ and the probability of the robot taking the horizontal edge is $\frac{1}{3}$.

III. If no edge at a vertex has been traced, then the probability of the robot taking the vertical edge is $\frac{2}{3}$ and the probability of the robot taking either of the horizontal edges is $\frac{1}{6}$.

(i) Given the robot starts from A, what is the probability it moves to B? **1**

(ii) Show that the probability of the robot taking the trail *ABCG* is $\frac{1}{27}$. **2**

(iii) List the six trails of length 3 from A to G. **1**

(iv) Determine the probability that the robot will trace a trail of length 3 from A to G. **2**

(v) Two robots attempt to travel on the cube at the same time and pace. Robot Alpha starts at A and robot Beta starts at G. Unfortunately, if they both want to use the same edge at the same time, then they crash and they cannot continue their trail. They cannot crash on the first edge of their trails. **3**

What is the probability of them ***not*** crashing on the second edge of their trails?

End of paper

2016 Year 12 Mathematics Trial SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question

MC:
1D
2A
3C
4C
5B
6D
7B
8D
9A
10C

1. $\frac{4}{3}\pi r^3 = 200$

$r = \sqrt[3]{\frac{600}{4\pi}}$

$= 3.6278316\ldots$

$r \approx 3.63$ (3sf) **D**

2. $m = \tan\alpha$

$m = \tan 120$

$= -\sqrt{3}$ **A**

3. $(-1)^k k^2$

$-1\times1^2 + (-1)^2\times2^2 + (-1)^3\times3^2 + (-1)^4\times4^2$

$= -1 + 4 - 9 + 16$

$= 10$

C

4. $1 - 4x - 5x^2 = 0$

$a = -5$, $b = -4$, $c = 1$

$\Delta = (-4)^2 - 4(-5)(1)$

$= 16 + 20$

$= 36$

$\therefore$ 2 real, rational roots

C

5. $\lim\limits_{h\to4} \frac{4-h}{16-h^2}$

$= \lim\limits_{h\to4} \frac{4-h}{(4-h)(4+h)}$

$= \lim\limits_{h\to4} \frac{1}{4+h}$

$= \frac{1}{8}$ **B**

6. $AB : BC = ED : DC$

$1:3 = ED:DC$

$DC = \frac{3}{4}\times 24$

$= 18$ **D**

7. $y = (x+1)^3(x-3)$

$u = (x+1)^3$ $v = x-3$

$u' = 3(x+1)^2$ $v' = 1$

$\frac{dy}{dx} = 3(x-3)(x+1)^2 + (x+1)^3$

$= (x+1)^2[3(x-3) + x+1]$

$= (x+1)^2(3x - 9 + x + 1)$

$= (x+1)^2(4x-8)$

$= 4(x+1)^2(x-2)$

B

8. $\int \frac{\sin x}{\cos x}\,dx$

$= -\int \frac{-\sin x}{\cos x}\,dx$

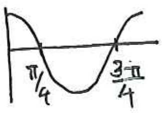
$= -\ln(\cos x) + C$

$= \ln \frac{1}{\cos x} + C$

$= \ln(\sec x) + C$

D

9. $\cos 2x(\tan x - 1) = 0$ $0 \le x \le \pi$



2 solutions

$\tan x = 1$

$x = \frac{\pi}{4}$ solution already accounted for

A

10. $\int_b^0 f(x)\,dx - \int_0^a f(x)\,dx$

C

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Suggested Solution (s)	Comments
Question 11	
a) $8x^3+27=(2x+3)(4x^2-6x+9)$ ✓	
b) $\frac{\sqrt{7}}{\sqrt{7}+2}\times\frac{\sqrt{7}-2}{\sqrt{7}-2}$ ✓	1 mark multiplying by correct conjugate
$=\frac{7-2\sqrt{7}}{7-4}$	
$=\frac{7-2\sqrt{7}}{3}$ ✓	1 mark correct answer
c) $y^2=-8x+24$ $y^2=-8(x-3)$ $a=2$ vertex $(3,0)$ ✓	1 mark rewriting equation or determining vertex
focus $(1,0)$ ✓	
d) $\frac{\cos x}{x^2}$ $u=\cos x$ $v=x^2$ $u'=-\sin x$ $v'=2x$ ✓	
$\frac{d}{dx}\left(\frac{\cos x}{x^2}\right)=\frac{-x^2\sin x-2x\cos x}{x^4}$ ✓	
$=\frac{-x\sin x-2\cos x}{x^3}$	do not need to simplify for full marks
e) $\frac{d}{dx}(e^x+x)^5=5(e^x+x)^4(e^x+1)$ $=5(e^x+1)(e^x+x)^4$ ✓✓	1 mark for attempting to use the chain rule 2 marks correct answer.
f) $\int_0^{\pi/2}\sin\left(\frac{x}{2}\right)dx=\left[-2\cos\frac{x}{2}\right]_0^{\pi/2}$ ✓	
$=-2\cos\frac{\pi}{4}--2\cos 0$ ✓	
$=-2\times\frac{1}{\sqrt{2}}+2\times 1$	
$=2-\frac{2}{\sqrt{2}}$ $=2-\sqrt{2}$ ✓	
g) $\int\frac{1}{x^2}dx=\int x^{-2}dx$	
$=-x^{-1}+C$ ✓ $=-\frac{1}{x}+C$ ✓	Must have '+c' for 2nd mark to be awarded.
h) $3\div 2=1{\cdot}5$ $3+1{\cdot}5=4{\cdot}5$ $a=4{\cdot}5$ ✓	

2016 Year 12 Mathematics Trial SOLUTIONS

Suggested Solution (s)	Comments

Question 12

a) $y = 4\sqrt{x}$ $(9, 12)$

$y = 4x^{\frac{1}{2}}$

$\frac{dy}{dx} = 2x^{-\frac{1}{2}}$

$= \frac{2}{\sqrt{x}}$ ✓

when $x = 9$

$\frac{dy}{dx} = \frac{2}{\sqrt{9}}$

$= \frac{2}{3}$ $\therefore$ gradient of tangent at $(9,12)$ is $\frac{2}{3}$

gradient of normal is $-\frac{3}{2}$ ✓

equation of normal:

$y - 12 = -\frac{3}{2}(x - 9)$

$2y - 24 = -3x + 27$

$3x + 2y - 51 = 0$ ✓ OR $y = -\frac{3}{2}x + \frac{51}{2}$

b) $5x^2 + 3x - 10 = 0$

$a = 5,\ b = 3,\ c = -10$

$\alpha + \beta = -\frac{3}{5}$

$\alpha\beta = \frac{-10}{5} = -2$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$ ✓

$= \left(-\frac{3}{5}\right)^2 - 2 \times (-2)$

$= \frac{9}{25} + 4$

$= \frac{109}{25} = 4.36$ ✓

c) i) $m_{AD} = \frac{1-3}{7-4}$

$= -\frac{2}{3}$

$y - 1 = -\frac{2}{3}(x - 7)$

$3y - 3 = -2x + 14$

$2x + 3y - 17 = 0$ as required

✓ Adequate working must be shown

ii) $d_{BC} = \sqrt{(6-3)^2 + (5-7)^2}$

$= \sqrt{9+4}$

$= \sqrt{13}$ ✓

iii) $d_{\perp} = \frac{|2\times 6 + 3\times 5 - 17|}{\sqrt{2^2+3^2}}$ ✓

$= \frac{|12 + 15 - 17|}{\sqrt{13}}$

$= \frac{10}{\sqrt{13}}$ ✓

iv) $A = \frac{10}{\sqrt{13}} \times \sqrt{13}$

$= 10$ units^2 ✓

d) $\int_2^6 \frac{x}{\ln x}\, dx$

x	2	4	6
$\frac{x}{\ln x}$	$\frac{2}{\ln 2}$	$\frac{4}{\ln 4}$	$\frac{6}{\ln 6}$

$\int_2^6 \frac{x}{\ln x}\, dx \doteq \frac{2}{3}\left(\frac{2}{\ln 2} + 4 \times \frac{4}{\ln 4} + \frac{6}{\ln 6}\right)$ ✓

$= 11.85040\ldots$

$\doteq 11.9$ (1dp) no penalty for rounding

2016 Year 12 Mathematics Trial SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 12

e) $A = 2\times\int_0^{\pi/2}(3\cos x - \sin 2x)\,dx$

$= 2\left[3\sin x + \frac{1}{2}\cos 2x\right]_0^{\pi/2}$ ✓

$= 2\left[(3\sin\frac{\pi}{2} + \frac{1}{2}\cos\pi) - (3\sin 0 + \frac{1}{2}\cos 0)\right]$ ✓

$= 2\left(\frac{5}{2} - \frac{1}{2}\right)$

$= 4 \text{ units}^2$ ✓

Alternate solution:

$A = \int_0^{\pi/2}(3\cos x - \sin 2x)\,dx + \int_{\pi/2}^{\pi}(\sin 2x - 3\cos x)\,dx$

$= \left[3\sin x + \frac{1}{2}\cos 2x\right]_0^{\pi/2} + \left[-\frac{1}{2}\cos 2x - 3\sin x\right]_{\pi/2}^{\pi}$ ✓

$= \left[(3\sin\frac{\pi}{2} + \frac{1}{2}\cos\pi) - (3\sin 0 + \frac{1}{2}\cos 0)\right]$
$+ \left[(-\frac{1}{2}\cos 2\pi - 3\sin\pi) - (-\frac{1}{2}\cos\pi - 3\sin\frac{\pi}{2})\right]$ ✓

$= \left[(3\times1 + \frac{1}{2}\times(-1)) - (3\times0 + \frac{1}{2}\times1)\right] + \left[(-\frac{1}{2}\times1 - 3\times0) - (-\frac{1}{2}\times(-1) - 3\times1)\right]$

$= \frac{5}{2} - \frac{1}{2} - \frac{1}{2} + \frac{5}{2}$

$= 4 \text{ units}^2$ ✓

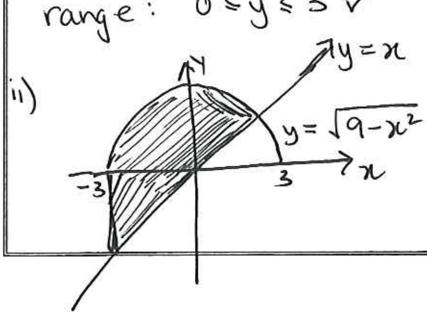
Question 13

a i) $f(x) = \sqrt{9 - x^2}$

domain: $-3 \le x \le 3$ ✓

range: $0 \le y \le 3$ ✓

ii)



1 mark for correct sketch of functions

1 mark for correct shading

b) $N(t) = 1200e^{kt}$ $k > 0$

$N(18) = 3000$

$N(0) = 1200$

i) $3000 = 1200e^{18k}$

$\frac{3000}{1200} = e^{18k}$

$\frac{5}{2} = e^{18k}$ ✓

Adequate steps of working must be shown.

$\ln\left(\frac{5}{2}\right) = \ln e^{18k}$

$\ln\left(\frac{5}{2}\right) = 18k$

$k = \frac{\ln\left(\frac{5}{2}\right)}{18}$

$= 0.0509050\ldots$

$\therefore k = 0.0509$ (4dp) ✓

ii) $2400 = 1200e^{0.0509t}$

$2 = e^{0.0509t}$

$\ln 2 = \ln e^{0.0509t}$ ✓

$0.0509t = \ln 2$

$t = \frac{\ln 2}{0.0509}$

$t = 13.6178\ldots$ ✓

$\therefore$ 14 years are required for the number of individuals to increase from 1200 to 1400 (correct to the nearest year)

2016 Year 12 Mathematics Trial SOLUTIONS

Suggested Solution (s)	Comments

Question 13

c) $f'(x) = 8x + 3$

$y = 4 - 5x \leftarrow$ tangent.

$M_{(\text{tangent})} = -5$

$-5 = 8x + 3$

$-8 = 8x$

$x = -1$ ✓

when $x = -1$

$y = 4 - 5\times(-1)$

$y = 9$

$\therefore$ point of contact $(-1, 9)$

$f'(x) = 8x + 3$

$f(x) = 4x^2 + 3x + c$ ✓

$f(-1) = 9 = 4(-1)^2 + 3(-1) + c$

$9 = 4 - 3 + c$

$8 = c$

$\therefore f(x) = 4x^2 + 3x + 8$ ✓

d) $y = \frac{1}{x+2}$

i) $y(x+2) = 1$

$(x+2) = \frac{1}{y}$

$x = \frac{1}{y} - 2$

$x^2 = \left(\frac{1}{y} - 2\right)^2$

$x^2 = \frac{1}{y^2} - \frac{4}{y} + 4$

Comment: Adequate steps of working must be shown for mark to be awarded.

ii) $V = \pi \int_a^b x^2\,dy$

$= \pi \int_{\frac{1}{2}}^{2} \left(\frac{1}{y^2} - \frac{4}{y} + 4\right) dy$

$= \pi \left[-\frac{1}{y} - 4\ln y + 4y\right]_{1/2}^{2}$ ✓

$= \pi \left[\left(-\frac{1}{2} - 4\ln 2 + 8\right) - \left(-2 - 4\ln 2^{-1} + 2\right)\right]$ ✓

$= \pi\left(7\tfrac{1}{2} - 4\ln 2 - 4\ln 2\right)$

$= \left(\frac{15}{2} - 8\ln 2\right)\pi$ units^3 ✓

Question 14a

$y = x^3 + 6x^2 - 135x$

i) $\frac{dy}{dx} = 3x^2 + 12x - 135$

$0 = 3x^2 + 12x - 135$

$0 = x^2 + 4x - 45$

$0 = (x+9)(x-5)$

$x = -9, 5$ ✓

when $x = -9$

$y = (-9)^3 + 6(-9)^2 - 135(-9)$

$= 972$ $\quad(-9, 972)$

when $x = 5$

$y = 5^3 + 6\times5^2 - 135\times5$

$= -400$ $\quad(5, -400)$ ✓

$\frac{d^2y}{dx^2} = 6x + 12$

when $x = -9$

$\frac{d^2y}{dx^2} = -42$ $\quad\therefore$ local maximum

when $x = 5$

$\frac{d^2y}{dx^2} = 42$ $\quad\therefore$ local minimum

$(-9, 972)$ Max, $(5, -400)$ minimum ✓

ii) $\frac{d^2y}{dx^2} = 6x + 12$

$0 = 6x + 12$

$-12 = 6x$

$x = -2$

$\therefore$ possible point of inflexion at $x = -2$

Check for change in concavity

x	-3	-2	-1
$\frac{d^2y}{dx^2}$	-6	0	6
	$\cap$	$\cdot$	$\cup$

Comment: ← required for mark to be awarded

$\therefore$ Change in concavity at $x = -2$

$\therefore$ point of inflexion at $x = -2$

2016 Year 12 Mathematics Trial SOLUTIONS

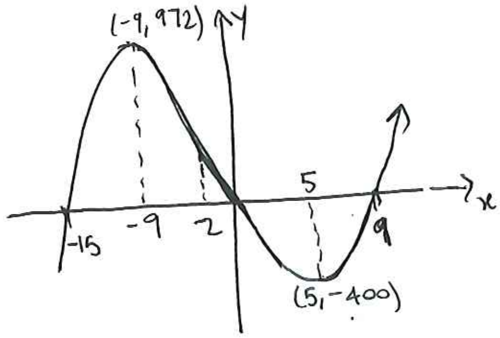
Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question

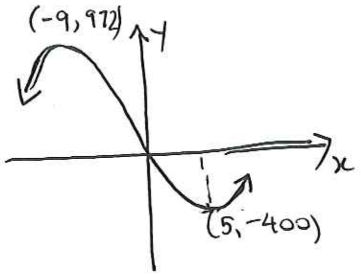
14a iii) $y = x^3 + 6x^2 - 135x$

$= x(x+15)(x-9)$

x intercepts at $0, -15, 9$



Also accept



Comments: 1 mark for correct shape

1 mark for correct labelling

x intercepts not required for full marks but must go through origin

14b) $R = 20 + \dfrac{100}{(1+2t)^2}$

i) $R = 20 + \dfrac{100}{(1+2\times 0)^2}$

$= 20 + \dfrac{100}{1}$

$= 120$ ✓

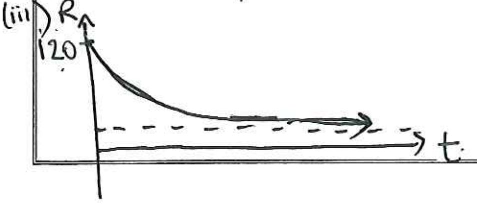
initially released at 120 tonnes/hour

ii) As $t \to \infty$

$R \to 20 + 0$

20 tonnes/hour ✓

(iii)



Comments: Must have asymptote and y intercept labelled/marked for the mark to be awarded.

iv) $\displaystyle\int_0^2 \left(20 + \frac{100}{(1+2t)^2}\right) dt$

$= \displaystyle\int_0^2 \left(20 + 100(1+2t)^{-2}\right) dt$

$= \left[20t + \dfrac{100(1+2t)^{-1}}{2\times(-1)}\right]_0^2$

$= \left[20t - \dfrac{50}{1+2t}\right]_0^2$ ✓

$= \left(20\times 2 - \dfrac{50}{5}\right) - \left(20\times 0 - \dfrac{50}{1}\right)$

$= 30 + 50$

$= 80$ tonnes ✓

c) $\dot{x} = 3 - \dfrac{9}{t-2}$

i) $t = 3$ $\quad \dot{x} = 3 - \dfrac{9}{3-2}$

$= -6$

$\therefore$ particle moving to the left ✓

(OR particle moving in the negative direction)

ii) let $\dot{x} = 0$

$0 = 3 - \dfrac{9}{t-2}$

$\dfrac{9}{t-2} = 3$

$9 = 3t - 6$

$15 = 3t$

$t = 5$ ✓ $\quad \therefore$ particle changes direction at $t = 5$ second

iii) distance $= \left|\displaystyle\int_3^5 3 - \frac{9}{t-2}\, dt\right| + \displaystyle\int_5^7 3 - \frac{9}{t-2}\, dt$ ✓

$= \left|\left[3t - 9\ln(t-2)\right]_3^5\right| + \left[3t - 9\ln(t-2)\right]_5^7$ ✓

$= |(15 - 9\ln 3) - (9 - 9\ln 1)| + [(21 - 9\ln 5) - (15 - 9\ln 3)]$

$= |6 - 9\ln 3| + 6 - 9\ln 5 + 9\ln 3$

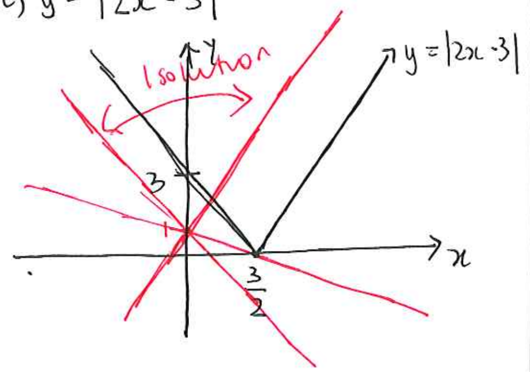
$= 5.29$ (2dp) ✓

2016 Year 12 Mathematics Trial SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 15

ai) $y = |2x - 3|$



1 mark for $y=|2x-3|$ with x and y intercept labelled.

ii) $m < -2$

$m \geq 2$

$m = -\frac{2}{3}$

2 mark all 3 sol'n identified correctly

1 mark for one or two of solutions identified

b) i) In ΔCED and ΔBAE

$\angle CED = \angle BAE$ given

$\angle CDE = \angle BEA$ corresponding angles $BE \parallel CD$ ✓

$\therefore \Delta CED \parallel\!\mid \Delta BAE$ equiangular ✓

ii) In ΔCEB and ΔEBA

$\angle CBE = \angle BAE$ given

$\angle BEC = \angle ECD$ alternate angles $BE \parallel CD$

$\angle ABE = \angle ECD$ matching angles in similar Δ (from i) ✓

$\therefore \angle BEC = \angle ABE$

$\therefore \Delta CEB \parallel\!\mid \Delta EBA$ equiangular ✓

Alternate solution:

ii) $\angle BAE = \angle CED$ given

$\therefore AB \parallel CE$ corresponding angles are equal with AD transversal

so $\angle ABE = \angle CEB$ alternate angles are equal with BE as transversal ✓

Consequently ΔEBA and ΔCEB are equiangular and hence similar. ✓

iii) $\frac{BE}{BA} = \frac{CE}{BE}$ sides of similar Δ are in ratio

$\frac{CE}{BE} = \frac{CD}{CE}$ sides of similar Δ are in ratio

$\therefore \frac{BE}{BA} = \frac{CE}{BE} = \frac{CD}{CE}$

$= \frac{x}{4} = \frac{y}{x} = \frac{13.5}{y} \Rightarrow \frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$

$\therefore 4, x, y, 13.5$ are 4 terms in geometric series

✓ appropriate working must be shown

iv) $4, x, y, 13.5$

$a = 4$

$ar^3 = 13.5$

$r^3 = 3.375$

$r = 1.5$ ✓

$x = 4 \times 1.5$
$= 6$

$y = 6 \times 1.5$
$= 9$ ✓

c) i) $A_1 = 500\,000 \times 1.004 - M$

$A_2 = A_1 \times 1.004 - M \times 1.01$

$= (500\,000 \times 1.004 - M) \times 1.004 - M \times 1.01$

$= 500\,000 \times 1.004^2 - M \times 1.004 - M \times 1.01$

$= 500\,000 \times 1.004^2 - M(1.004 + 1.01)$

adequate working must be shown

ii) $A_4 = 500000 \times 1.004^4 - M(1.004^3 + 1.004^2(1.01) + 1.004(1.01)^2 + 1.01^3)$

$A_{60} = 500\,000 \times 1.004^{60} - M(1.004^{59} + 1.004^{58}(1.01) + 1.004^{57}(1.01)^2 + \ldots + 1.01^{59})$ ✓

iii) $A_{60} = 400\,000$

$400\,000 = 500000 \times 1.004^{60} - M(1.004^{59} + 1.004^{58}(1.01) + 1.004^{57}(1.01)^2 + \ldots + 1.01^{59})$

$a = 1.004^{59}$ $\quad r = \frac{1.01}{1.004}$ $\quad n = 60$

$400\,000 = 500\,000 \times 1.004^{60} - M\left[\frac{1.004^{59}\left(\left(\frac{1.01}{1.004}\right)^{60} - 1\right)}{\left(\frac{1.01}{1.004} - 1\right)}\right]$ ✓

$400000 \times \left(\frac{0.006}{1.004}\right) = 500\,000 \times 1.004^{60} \times \left(\frac{0.006}{1.004}\right) - M\left(1.004^{59}\left(\left(\frac{1.01}{1.004}\right)^{60} - 1\right)\right)$

$M = \frac{500\,000 \times 1.004^{60} \times \frac{0.006}{1.004} - 400\,000 \times \frac{0.006}{1.004}}{1.004^{59}\left(\left(\frac{1.01}{1.004}\right)^{60} - 1\right)}$ ✓

$M = \$2585.67$ (nearest cent)

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Suggested Solution (s)	Comments	Suggested Solution (s)	Comments
Question 16 ai) $\cos\theta = \frac{1}{MN}$ $\therefore MN = \frac{1}{\cos\theta}$ $MN = \sec\theta$ $\tan\theta = \frac{MT}{1}$ $MT = \tan\theta$ } 1 mark. ii) Total cost = Cost LM + Cost MN $T = d_{LM} \times c + 3c \times d_{MN}$ $d_{LM} = 3 - MT$ $= 3 - \tan\theta$ $d_{MN} = \sec\theta$ ✓ $T = (3-\tan\theta)c + 3c(\sec\theta)$ $= 3c - c\tan\theta + 3c\sec\theta$ ✓ iii) $\frac{dt}{d\theta} = 0$ for minimised cost $0 < \theta < \pi/2$ $T = 3c - c\tan\theta + 3c\sec\theta$ $= 3c - c\tan\theta + 3c(\cos\theta)^{-1}$ $\frac{dT}{d\theta} = -c\sec^2\theta + -3c(\cos\theta)^{-2}(-\sin\theta)$ $= -c\sec^2\theta + \frac{3c\sin\theta}{\cos^2\theta}$ ✓ $= -c\sec^2\theta + 3c\sin\theta\sec^2\theta$ $= c\sec^2\theta(-1 + 3\sin\theta)$ $0 = c\sec^2\theta(3\sin\theta - 1)$ $c\sec^2\theta = 0$ No solution; $3\sin\theta - 1 = 0$, $\sin\theta = \frac{1}{3}$ ✓		Check it is a minimum Radians θ: 0.3, 0.339..., 0.4 $\frac{dT}{d\theta}$: −0.124, 0, 0.1983... \ — / ✓ ∴ minimum occurs at 0.3398... radians If used degrees θ: 19, 19.42..., 20 $\frac{dT}{d\theta}$: −0.026..., 0, 0.029... \ — / ∴ An angle of 0.34 radian (to 2 dp) would minimise the cost.	

$\theta = 0.3398...$ radians

(19.47122°)

2016 Year 12 Mathematics Trial SOLUTIONS

Suggested Solution (s)	Comments	Suggested Solution (s)	Comments

Question 16

bi) $\frac{1}{6}$ ✓

ii) $AB \rightarrow \frac{1}{6}$

then $BC \rightarrow \frac{1}{3}$

then $CG \rightarrow \frac{2}{3}$ ✓

$P(ABCG) = \frac{1}{6} \times \frac{1}{3} \times \frac{2}{3}$

$= \frac{1}{27}$ ✓

iii) ABCG, ABFG, ADCG, ADHG, AEHG, AEFG — 6 trails ✓

iv) $ABCG = \frac{1}{27}$ HHV

$ABFG = \frac{1}{6} \times \frac{2}{3} \times \frac{1}{2}$ HVH

$= \frac{1}{18}$

$ADCG = \frac{1}{27}$ HHV

$ADHG = \frac{1}{18}$ HVH

$AEHG = \frac{2}{3} \times \frac{1}{2} \times \frac{1}{3}$ VHH

$= \frac{1}{9}$

$AEFG = \frac{1}{9}$ VHH

$P(\text{length } 3 \text{ A to G}) = \frac{1}{27} + \frac{1}{18} + \frac{1}{27} + \frac{1}{18} + \frac{1}{9} + \frac{1}{9}$

$= \frac{2}{27} + \frac{2}{18} + \frac{2}{9}$

$= \frac{11}{27}$

v) They can only crash on the second edge of their trails and so the second and third letters of a trail need to be the same but reverse order for a crash to occur.

$AEH = \frac{1}{6} \times \frac{2}{3} = \frac{1}{9}$	$GHE = \frac{1}{6} \times \frac{2}{3} = \frac{1}{9}$
$ADC = \frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$	$GCD = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$
$ABF = \frac{1}{6} \times \frac{2}{3} = \frac{1}{9}$	$GFB = \frac{1}{6} \times \frac{2}{3} = \frac{1}{9}$
$ABC = \frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$	$GCB = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$
$AEH = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$	$GHE = \frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$
$AEF = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$	$GFE = \frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$

✓

$P(\text{crashing on second edge})$

$= \frac{1}{9} \times \frac{1}{9} + \frac{1}{18} \times \frac{1}{3} + \frac{1}{9} \times \frac{1}{9} + \frac{1}{18} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{18} + \frac{1}{3} \times \frac{1}{18}$ ✓

$= \frac{1}{81} \times 2 + \frac{1}{54} \times 4$

$= \frac{8}{81}$

$P(\text{not crashing on second leg})$

$= 1 - \frac{8}{81}$

$= \frac{73}{81}$ ✓